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LETTER TO THE EDITOR

On the problem of particle–hole symmetry in even–even Cd and Te nuclei

Gertrude Scharff-Goldhaber
Brookhaven National Laboratory, Upton, New York 11973, USA

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Abstract. A comparison of the pattern of level energies of the ground-state bands in even–even Cd and Te nuclei indicates that in Te nuclei for which proton and neutron orbits belong to the *same* major shell (50 → 82), pronounced neutron–proton interaction occurs. This explains the large $h_{11/2}$ admixture to the proton configuration of the neutron rich Te isotopes underlying the negative quadrupole moments commented on by Hall and Thomas.

The purpose of this letter is to demonstrate that a comparison of the pattern of level energies of ground-state bands $(2^+, 4^+, 6^+, \ldots)$ in the Cd and Te isotopes gives strong evidence of configuration mixing in Te nuclei, brought about by neutron–proton interactions.

Some aspects of these patterns were discussed recently by Sakai (1973). This led Hall and Thomas (1974) to point out that while the energy levels of the lowest 2^+ states in Cd ($Z = 48$) and Te ($Z = 52$) isotone pairs are quite similar and thus appear to indicate the presence of a $(g_{9/2})^{-2}$ and $(g_{7/2})^2$ proton configuration respectively, the sign of the static quadrupole moments in the neutron-rich Te nuclei ($N = 70 \rightarrow 78$) implies that *higher* proton orbitals contribute significantly. They conclude that 'the quadrupole moment and the level energy have a very different sensitivity to the details of the nuclear wavefunction'.

In fact, by examining more fully the energy spectra discussed by Sakai, we shall see that they reveal even more information about configuration mixing than do the static quadrupole moments.

Studies of first excited states of even–even nuclei, carried out more than twenty years ago, showed that E_2, the energy of the lowest 2^+ state, is a smooth function of N and Z, and is closely related to shell structure. Soon afterward A de Shalit and M Goldhaber (1953) suggested that neutron–proton interactions may contribute importantly to the magnitude of E_2. They pointed out that the 'valence' nucleons of one kind (neutron or proton) with given quantum numbers n and l may have the most 'stabilizing' influence on valence nucleons of the other kind with l' and n', if $l-l'$ and $n-n'$ vanish or at least attain the smallest possible values. They gave several examples for their rule, all of which concerned near-magic nuclei. This 'overlap principle' yields approximate predictions for configuration mixing.

So far, 'full' shell-model calculations for medium heavy and heavy nuclei have—with the exception of a few particularly simple cases—not taken neutron–proton interactions into account. This may be one of the reasons why such calculations usually cannot 'predict' experimental E_2 and E_4 values. Hence, one is still restricted to a phenomenological approach.

The following analysis of GS band energies is based on the variable moment of inertia (VMI) law (G Scharff-Goldhaber and A S Goldhaber 1970), which relates the higher states ($J \geqslant 6$) of a ground state band to the 'scale factor' E_2 and the energy ratio $R_4 = E_4/E_2$. Figure 1(*a*) presents E_2 plotted against N for the Cd ($Z = 48$) and Te ($Z = 52$) isotopes, and, as background information, for the Pd ($Z = 46$), Sn ($Z = 50$), Xe ($Z = 54$) isotopes. The values for the $N = 80$, 82 Te nuclei, not included in Sakai's figure, are taken from Wilhelmy *et al* (1970) and Kerek (1971), and the value for the $N = 50$ Pd nucleus is taken from Piel *et al* (1974). This figure shows that the E_2 values for Te nuclei are consistently higher than for Cd nuclei, while the 'four-particle' Xe and 'four-hole' Pd curves approximately coincide—and slightly cross—in the region where they overlap.

In figure 1(*b*), R_4 is plotted against N, again for $Z = 46$ (Pd) to 54 (Xe). In general, the ratio R_4 is a roughly monotonic function of the number of proton and neutron pairs (particles or holes) within a shell; it ranges between $1 < R_4 < 1{\cdot}82$ for doubly or

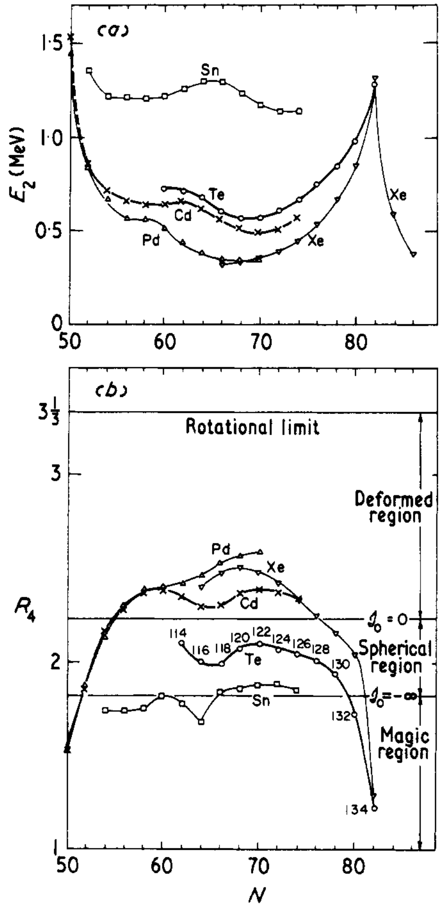

Figure 1. (*a*) Energies (in MeV) of first 2^+ states, plotted against neutron number, for Cd ($Z = 48$) and Te ($Z = 52$) (bold curves), and Sn ($Z = 50$), Pd ($Z = 46$), Xe ($Z = 54$) (thinner curves). (*b*) R_4 ($= E_4/E_2$) values, plotted against neutron number, for the same nuclear species as in (*a*). The maxima in the curves in figure 1(*a*) coincide approximately with minima in (*b*). In both figures these extrema are barely visible in the Pd and Xe curves.

singly magic nuclei (eg, R_4 (^{208}Pb) = 1·05), between $1{\cdot}82 < R_4 \leqslant 2{\cdot}23$ for 'spherical' nuclei, and between $2{\cdot}23 \leqslant R_4 < 3{\cdot}33$ for 'deformed' nuclei. At $R_4 = 2{\cdot}23$, the VMI parameter $\mathscr{J}_0$ vanishes, and the softness parameter $\sigma = 1/2C\mathscr{J}_0^3$ becomes infinite. In other words, at $R_4 = 2{\cdot}23$, the deforming forces just balance the forces which bring about a rigid spherical shape. For $R_4 > 2{\cdot}23$, the 'ground state moment of inertia' $\mathscr{J}_{J=0}$ equals $\mathscr{J}_0$, whereas for $R_4 < 2{\cdot}23$, $\mathscr{J}_{J=0} = 0$. As R_4 decreases from 2·23 to 1·82, the increasing negative value of $\mathscr{J}_0$ provides a measure for the resistance to cranking which the nucleus experiences: it has to overcome the threshold energy $\frac{1}{2}C\mathscr{J}_0^2$. At $R_4 = 1{\cdot}82$, $\mathscr{J}_0 \rightarrow -\infty$. Below this point, cranking is only possible if the nuclear matter is rearranged, ie if some nucleons, eg two or four, are promoted to higher orbits. The magic region is outside the limit of validity of the VMI law.

It is immediately apparent, as was already noted by Sakai, that a significant difference exists between Cd and Te nuclei with respect to R_4: all Cd nuclei with the exception of ^{100}Cd$_{52}$ and ^{102}Cd$_{54}$, are deformed, whereas the Te nuclei, up to $N = 78$, are spherical. Moreover, the nucleus ^{132}Te ($N = 80$) even violates the empirical rule given above; its R_4 value lies in the magic zone. Similar to the R_4 curve for the magic species $Z = 50$ (Sn) which has a minimum at $N = 64$, the $Z = 48$ (Cd) and 52 (Te) curves exhibit minima at $N = 64$ through 66. However, for the almost coinciding R_4 curves of the more deformed species $Z = 46$ and 54 the minima have almost disappeared.

It is also interesting to examine for Cd and Te nuclei the angular momentum boundary of the VMI phase (Scharff-Goldhaber 1972, 1974), ie of the function $J_{\text{crit}}(Z, N)$, above which the GS bands deviate from the VMI law. In figure 2 the logarithms of the ratios

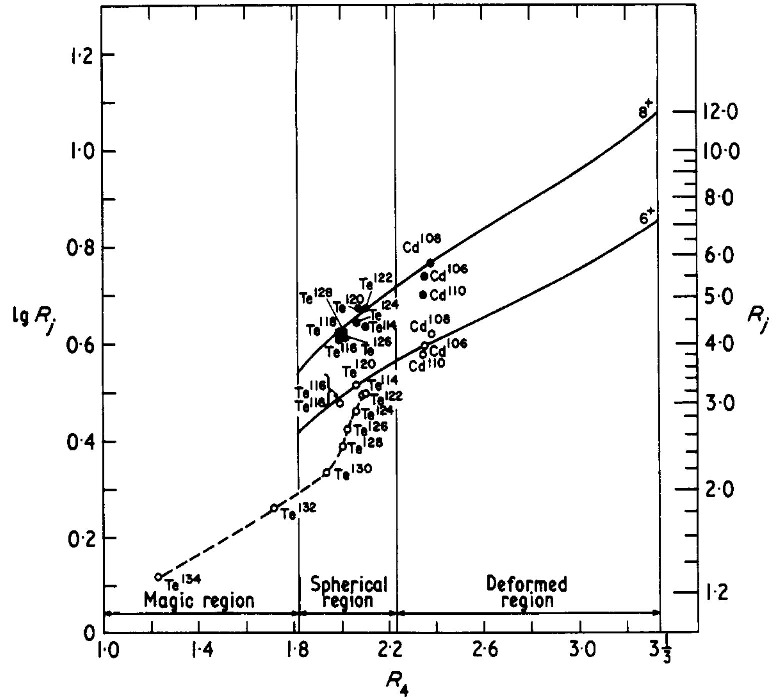

Figure 2. The full curves show the VMI predictions for R_6 and R_8 against R_4. The circles indicate the measured values for these quantities. The R_6 values (open circles) for Te are seen to deviate more and more from the predicted values as N approaches 82. The deviations correspond to increasing 'backbending' of the respective $\mathscr{J}$ against ω^2 curves.

$R_J = E_J/E_2$ are plotted against R_4. The VMI solutions for R_6 and R_8 are shown as full curves. It is seen that for $^{106\to110}$Cd and for $^{114\to122}$Te, R_6 (open circles) and R_8 (full circles) lie fairly close to the VMI curves. However, for ^{124}Te → ^{130}Te, a systematic downward deviation of R_6 occurs, ie for these nuclei, $J_{\text{crit}} = 4$ (VMI is violated immediately) whereas for the nuclei with $N < 70$, $J_{\text{crit}} \geqslant 8$. The R_8 values for ^{124}Te → ^{128}Te lie again closer to the VMI curve.

We shall now attempt to interpret these phenomena: the minima in the Cd, Sn, and Te curves in figure 1(*b*) appear to be due to subshell closing of the $1g_{7/2}(8)$ and $2d_{5/2}(6)$ orbits. The somewhat larger E_2 and considerably smaller R_4 values for Te compared to Cd are most naturally ascribed to a large admixture to the $g_{7/2}$ proton orbits of those orbits in the 50 to 82 shell which show the highest overlap with neutron orbits. In particular, for ^{132}Te the overlap for $(h_{11/2})^2$ protons with the $h_{11/2}$ neutrons (of which there may be 10 or 12) is very strong: hence the 'quasi-magic' R_4 value.

The angular momentum boundary or 'phase transition' illustrated in figure 2 has been attributed to degeneracy of two orbits (Sorensen 1973), leading to configuration mixing. It occurs at the lowest J_{crit} for Te nuclei with neutrons in the $h_{11/2}$ orbits, for which, as we saw above, the protons prefer to occupy the same orbits. The increasing reduction in the $4^+ \to 6^+$ level spacing as the neutron shell is filled, corresponds to an increase in 'backbending' of the $\mathscr{I}$ against ω^2 curves (see Sorensen 1973), with following 'forward bending' between the 6^+ and 8^+ states.

Sakai's finding that also at other closed proton and neutron shells the R_4 values for the two-hole nuclei exceed those for two-particle nuclei leads to the expectation that neutron–proton interaction with resulting strong configuration mixing for *two-particle* nuclei is a general phenomenon.

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